

# Towards a spin foam model description of black hole entropy

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## Abstract

We propose a way to describe the origin of black hole entropy in the spin foam models of quantum gravity. This stimulates a new way to study the relation of spin foam models and loop quantum gravity.

## 1 Introduction

Derivation of the Bekenstein-Hawking black hole entropy [1], [2] from a quantum theory of gravity is one of the most important problems of physics. It is a must for any theory of quantum gravity to give a satisfactory statistical description. Here we propose a way to describe black hole entropy in a theory of quantum gravity known as spin foam models. Spin foam models are the covariant version of loop quantum gravity.

Loop quantum gravity is a very promising theory which has studied the statistical description of black hole entropy where seminal ideas come from [3], [4]. Later, the problem has been developed deeper following different

approaches for the counting of the degrees of freedom [5], [6], [7], [8], [9], [10]. The problem has even been extended to horizon surface of higher genus [11].

The problem of giving a statistical description of the entropy in terms of spin foam models has been considered for the three dimensional BTZ black hole only [12]. Here we present a proposal to study the entropy of a four dimensional spherically symmetric black hole in terms of spin foam models. The study of black hole entropy could be a way to relate loop quantum gravity to spin foam models or at least it can give more evidence of the relation.

We divide this letter as follows: In section 2 we describe very briefly the

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idea behind the statistical description of black hole entropy in loop quantum gravity. In section 3 we propose a way to study the description of black hole entropy in spin foam models.

## 2 Black hole entropy in loop quantum gravity

Let us consider the basic ideas behind black hole entropy in loop quantum gravity. Consider the surface of a Schwarzschild black hole. In loop quantum gravity its geometry is determined by the punctures of the edges of a spin network embedded in three dimensional space with the surface. Label the edges of the spin network by  $j_i$  which are half integers, that is, irreducible representations of the group  $SU(2)$ . Suppose the spin network punctures the surface in  $n$  isolated points, and in a non-degenerate way. The total area of the surface is given by the eigenvalues of the area operator  $A$

$$A = 8\pi\gamma l_p^2 \sum_{i=1}^n \sqrt{j_i(j_i + 1)} \quad (1)$$

where  $l_p$  is the Planck length and  $\gamma$  is a parameter known by the name of Immirzi.

The entropy of the black hole is given by the logarithm of the number  $N$  of states which account for a fixed area of the surface

$$S = \log N \quad (2)$$

The number  $N$  of these states has been considered in [5], [6], [7], [8], [9], [10]; however the states which account for the entropy vary in opinions and we have various possibilities.

The question is whether we can get a counting related to the one in loop quantum gravity by using the spin foam models of quantum gravity.

## 3 Black hole entropy from spin foam models

In this section we propose a way to compute the entropy of a spherically symmetric black hole in spin foam models of quantum gravity. In this letter we will just give a description of our idea and the important calculations will appear in a future paper.

In spin foam models we think of space-time as discrete. It is thought for instance as a 2-complex dual to a triangulation of the space-time manifold, or as the triangulation itself. In [12] we considered the entropy of BTZ black hole from a spin foam model viewpoint. Here we do something similar. Consider a spherical symmetric black hole such as Schwarzschild. Take a three dimensional subspace such as a Cauchy surface in which the spherical surface of the horizon is embedded. This is, we take  $S^2$  embedded in  $R^3$ . Decompose  $S^2$  as a 2-complex. Such decomposition will be of course part of the 2-complex which represents the whole space-time.

Consider a 2-complex decomposition of the horizon surface  $S^2$ , one

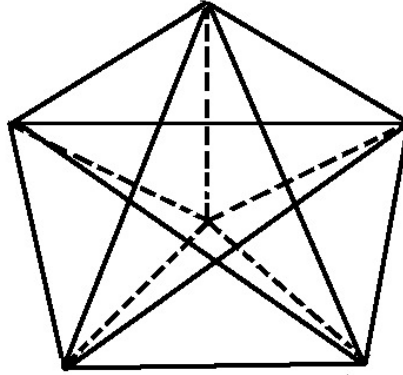


Figure 1: 4-simplex subdivision

which is dual to a non-degenerate triangulation. We can think that the non-degenerate triangulation of the sphere  $S^2$  was obtained by subdivisions of 4-simplexes as figure [1]. The edges of the graph of figure [1] correspond to faces of the dual 2-complex. We can also think that three of the dotted lines of the figure correspond to faces of the horizon surface.

All the remaining faces of the horizon surface will be part of other subdivided 4-simplexes.

Consider a spin foam state sum where faces of the 2-complex are labelled by spins which are irreducible representations of a group or a quantum group. The sum is over admissible irreducible representations of the product of amplitudes given to faces and vertices.

We will take the spins labelling the faces of the horizon surface of the black hole as fixed. Denote the horizon surface as  $\mathcal{O}$ . Consider the state sum

$$Z(M, \mathcal{O}) = \sum_{S|\mathcal{O}} \prod_{\text{faces}} \dim_q(j) \prod_{\text{vertices}} A_v \quad (3)$$

where  $M$  is the 2-complex which represents the space-time,  $A_v$  is the amplitude of vertices,  $S|\mathcal{O}$  means that the irreducible representations which label the horizon faces are not summed over. Clearly the state sum is a function of the labels of the horizon.

In the dictionary of loop quantum gravity versus spin foam models we propose that the punctures of the spin network with the horizon surface correspond in spin foam models to this description of having fixed spins at the horizon in the state sum.

We define the entropy of the black hole as

$$\log \left( \frac{Z(M, \mathcal{O})}{Z(M)} \right) \quad (4)$$

where  $Z(M)$  is the state sum of the space-time manifold without fixed

spins at the horizon. The idea now is to prove that our definition of the entropy is related to the entropy computed in loop quantum gravity. The important question now is which state sum will get the entropy of loop quantum gravity. Alternatively we ask, which spin foam model is the one which will lead us to an entropy related to the one of loop quantum gravity. We have the BC model [13], and two new models which are more promising [14],[15].

It could be very interesting to study formula (4) for these models to see whether it is related to the calculation of black hole entropy in loop quantum gravity. In a future paper we will compute formula (4) for the case of Riemannian BC model with cosmological constant, which means that we will use the quantum group  $SO_q(4)$ .

## References

- [1] J.D. Bekenstein, Black Holes and Entropy, Phys. Rev. D. **7** (1973) 2333-2346
- [2] S.W. Hawking, Particle Creation by Black Holes, Commun. Math. Phys **43** (1975) 199-220
- [3] K. Krasnov, Geometrical entropy from loop quantum gravity, Phys. Rev. **D55** (1997) 3505
- [4] Carlo Rovelli, Black Hole Entropy from Loop Quantum Gravity, Phys.Rev.Lett. **77** (1996) 3288-3291. arXiv:gr-qc/9603063v1
- [5] A. Ashtekar, J. Baez, A. Corichi and K. Krasnov, Quantum geometry and black hole entropy, Phys. Rev. Lett. **80** (1998) 904-907. arXiv:gr-qc/9710007
- [6] M. Domagala and J. Lewandowski, Black hole entropy from quantum geometry, Class. Quant. Grav. **21** (2004) 5233-5244. arXiv:gr-qc/0407051
- [7] A. Corichi, J. Diaz-Polo and E. Fernandez-Borja, Quantum geometry and microscopic black hole entropy, Class. Quant. Grav. **24** (2007) 243-251. arXiv:gr-qc/0605014
- [8] T. Tamaki, Considering boundary conditions for black hole entropy in loop quantum gravity, Class. Quant. Grav. **24** (2007) 3837-3842. arXiv:0707.0341 [hep-th].
- [9] Ivan Agullo, J. Fernando Barbero, Jacobo Diaz-Polo, Enrique Fernandez-Borja, Eduardo J. S. Villaseor, Black hole state counting in Loop Quantum Gravity: A number theoretical approach, Phys. Rev. Lett. **100** (2008) 211301. arXiv:0802.4077v2 [gr-qc]
- [10] T.Tanaka, T.Tamaki, Black hole entropy for the general area spectrum. arXiv:0808.4056v1 [hep-th]
- [11] S. Kloster, J. Brannlund, A. DeBenedictis, Phase-space and Black Hole Entropy of Higher Genus Horizons in Loop Quantum Gravity, Class. Quantum Grav. **25** (2008) 065008. arXiv:gr-qc/0702036v3
- [12] J.Manuel Garcia-Islas, BTZ Black Hole Entropy: A spin foam model

- p description, arXiv:0804.2082v1
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- [gr-qc]
- [13] John W. Barrett, Louis Crane, Relativistic spin networks and quantum gravity, J.Math.Phys. **39** (1998) 3296-3302. arXiv:gr-qc/9709028v2
- [14] Laurent Freidel, Kirill Krasnov, A New Spin Foam Model for 4d Gravity, Class. Quantum Grav. **25** (2008) 125018. arXiv:0708.1595v2 [gr-qc]
- [15] Jonathan Engle, Roberto Pereira, Carlo Rovelli, The loop-quantum-gravity vertex-amplitude, Physical Review Letters **99** (2007) 161301. arXiv:0705.2388v1 [gr-qc]